Self-Consistent Correlations of Randomly Coupled Rotators in the Asynchronous State

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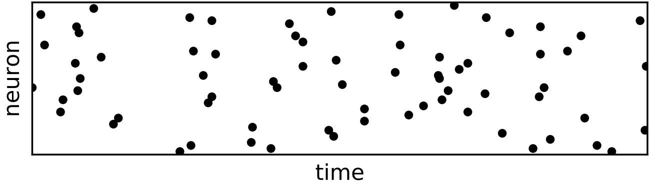
Introduction

Spiking activity of cortical neurons in behaving animals is highly *irregular and asynchronous* (e.g. Harris & Thiele 2011):



Quasi stochastic activity (network noise) arises most likely from *nonlinear chaotic interactions* in the network.

Aim: analytically tractable toy model of asynchronous state

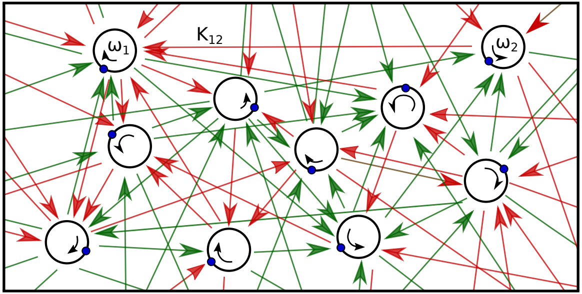
Model

State:

$$x_m(t) = e^{i\Theta_m(t)}$$

Dynamics:

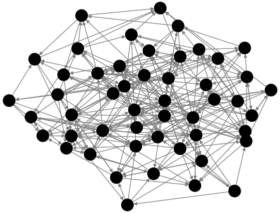
$$\dot{\Theta}_m = \omega_m + \sum_{n \neq m} K_{mn} f(\Theta_n)$$



- i.i.d. natural frequencies ω_m
- i.i.d. coupling coefficients K_{mn} with $\langle K_{mn} \rangle_K = \bar{K}/N$, $\langle \Delta K_{mn}^2 \rangle_K = K^2/N$
- 2π-periodic interaction function $f(\Theta_n)$

Approach

Self-consistent theory of network fluctuations and single unit correlation function:



- recurrent input → effective network noise
- input and output fluctuations intricately related → self-consistent description
- statistics of interest contained in temporal autocorrelations → power spectra

- pioneered in the seminal work of Sompolinsky, Crisanti, Sommers (1988)
- considerable recent interest (Kadmon & Sompolinsky 2015; Schuecker, Goedeke, Helias 2018; Mastrogiuseppe & Ostojic 2018; Muscinelli, Gerstner, Schwalger 2019)

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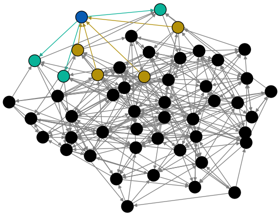

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Theory

Stochastic mean-field approximation

Dynamics:

$$\dot{\Theta}_m = \omega_m + \sum_{n \neq m} K_{mn} f(\Theta_n)$$

Recurrent input approximated by i.i.d. Gaussian noise processes $\xi_m(t)$:

$$\dot{\Theta}_m = \omega_m + \xi_m(t)$$

$$\mu_\xi(t) = \bar{K} \left\langle f(\Theta(t)) \right\rangle_{\xi,\omega}$$

$$C_\xi(t, t') = K^2 \left\langle f(\Theta(t)) f(\Theta(t')) \right\rangle_{\xi,\omega}$$

Self-consistent statistics

Stationarity, rotation-invariance:

$$\ddot{\Lambda} = K^2 \sum_{\ell=-\infty}^{\infty} |A_\ell|^2 \, \phi(\ell\tau) e^{-\ell^2 \Lambda}$$

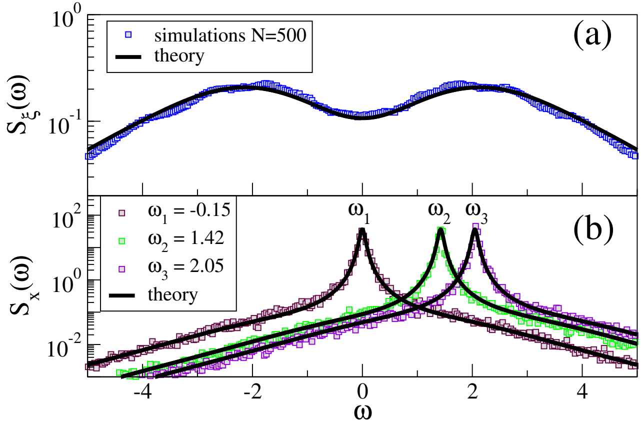
All statistics of interest follow:

$$C_\xi(\tau) = K^2 \sum_{\ell=-\infty}^{\infty} |A_\ell|^2 \, \phi(\ell\tau) e^{-\ell^2 \Lambda(\tau)}$$

$$C_{x_m}(\tau) = e^{i\omega_m \tau - \Lambda(\tau)}$$

Validation (1)

Network noise spectrum $S_\xi(\omega)$ and single unit spectrum $S_x(\omega)$:



Distributed natural frequencies:

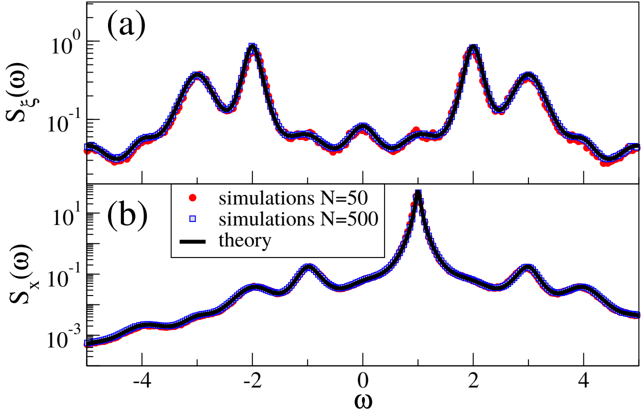
$$\omega_m \sim \mathcal{N}(1, 1/2)$$

$$\bar{K} = 0, \quad K = 1/2$$

$$f(\Theta) = \cos(2\Theta) + \sin(3\Theta)$$

Validation (2)

Network noise spectrum $S_\xi(\omega)$ and single unit spectrum $S_x(\omega)$:



Unique natural frequency:

$$\omega_m = 1$$

$$\bar{K} = 0, \quad K = 1/2$$

$$f(\Theta) = \cos(2\Theta) + \sin(3\Theta)$$

Extension to Spiking Networks

Mimicking spikes arising from threshold crossings in integrate-and-fire neurons:

$$\dot{\Theta}_m = \omega_m + \sum_{n \neq m} K_{mn} \dot{\Theta}_n \sum_{k=-\infty}^{\infty} \delta(\Theta_n - 2\pi k)$$

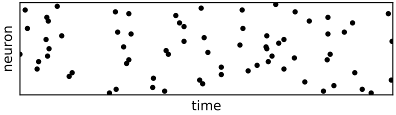
Self-consistency equation for
network statistics in stationary, rotation-invariant state:

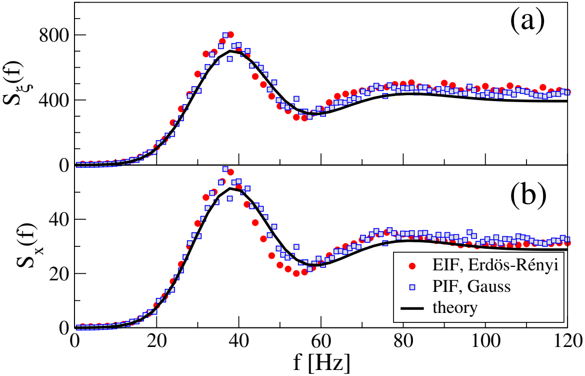
$$\ddot{\Lambda} = \frac{K^2}{4\pi^2} \{ (\ddot{\Lambda} + \sigma_\omega^2 + \omega_0^2) \Psi(\Lambda, \tau) + \omega_0 (\sigma_\omega^2 \tau + \dot{\Lambda}) \Psi'(\Lambda, \tau) + \tfrac{1}{4} (\sigma_\omega^2 \tau + \dot{\Lambda})^2 \Psi''(\Lambda, \tau) \}$$

$$\text{with} \quad \Psi(\Lambda, \tau) = \vartheta_3[\tfrac{1}{2}\omega_0 \tau, e^{-\frac{1}{2}\sigma_\omega^2 \tau^2 - \lambda^2 - \Lambda}]$$

Mean-Driven Network of Spiking Neurons

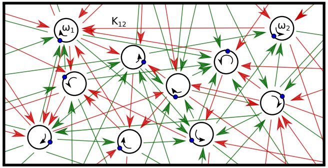
Sparse heterogeneous network of
excitatory and inhibitory exponential integrate-and-fire (EIF) neurons:

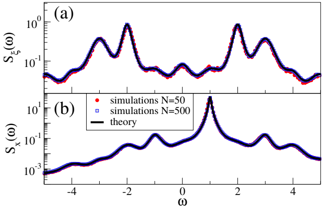


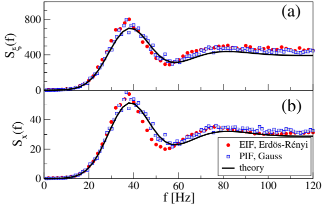


- Dale's law
- cell-to-cell variability
- mean-driven regime

Summary







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Poster: P212 (Monday)